SUBTRACTION: Y1

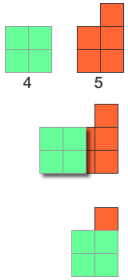
Understanding the operation and vocabulary

Understanding the operation

understand subtraction as:

'taking away' - removing part of a set & reduction

'difference' – comparison & how much more is needed



Identify one less than a given number

Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs ;

14 – 3 = 11 9 = 16 - 7

Solve missing number problems and recognise use of inverse e.g.

11 - ☐ = 8 ☐ = 13 - 2 3 = ☐ - ☐

Realise the effect of adding or subtracting 0

Establish addition and subtraction as related operations.

19 +1 = 20 so 20 - 1 = 19

Vocabulary

Understand the vocabulary related to subtraction.

subtraction, subtract, take away, minus, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit

Generalisations

- True or false? Subtraction makes numbers smaller
- When introduced to the equals sign, pupils should see it as signifying equality. They should become used to seeing it in different positions.

Understanding the operation and vocabulary

Number facts

Recall and use subtraction facts to 10 fluently e.g.

6 minus 3 8 subtract 2 4 less than 9

Know number pairs with a total of 20 and derive related subtraction facts e.g.

20 + 0, 20 - 1, 20 - 2, 20 - 3 …

Memorise and reason with number bonds to 10 and 20 in several forms

9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9; 9 = 16 – 7 etc.

6 – 4 = 2 so 16 - 4 = 12

Mental methods and jottings

Subtract one-digit and two-digit numbers to 20, including zero, using apparatus including number lines.

Represent and use number bonds within 20

Partition a given number of objects (up to 20) into 2 groups e.g.

Partition 15 into 7 and 8, 9 and 6 ….

Counting back

15 - 3 (by counting back 3 in ones; 14, 13, 12)

Progress to crossing the tens boundary

With jottings

15 – 6 (by counting back in ones or partitioning 6 to bridge the tens boundary; -5, -1)

Counting up

9 – 6 (by counting up from 6 to 9 in ones; 7, 8, 9)

With jottings

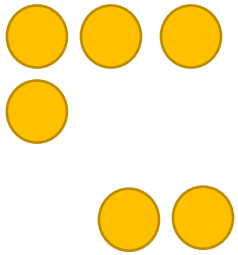
19 – 14 (by counting up from 14 to 19 in ones; 15, 16, 17, 18, 19)

Understanding the operation and vocabulary

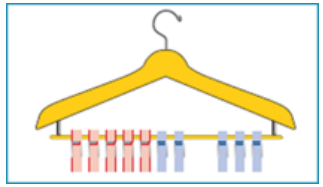
https://www.ncetm.org.uk/resources/50640

CONCRETE

Use physical objects, counters, cubes etc to show how objects can be taken away.

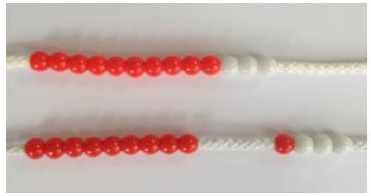


6 – 2 = 4



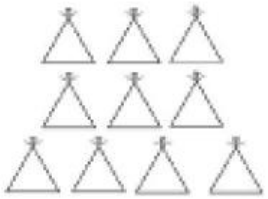
Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

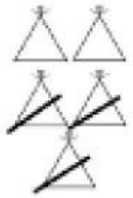
13 – 4



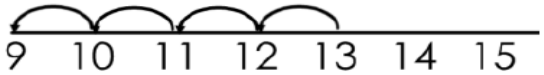
PICTORIAL

Cross out drawn objects to show what has been taken away.





15 – 3 = 12

Misconceptions • Pupils struggle to interpret whether to add or subtract from the language used. • Pupils often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this. • The equals sign is not always correctly interpreted as 'has the same value as' by pupils who see it as 'the answer is'	Know doubles to at least 10 and use near doubles to add pairs of numbers **Using known facts and place value** 6 – 4 = 2 so 16 - 4 = 12 **Using known doubles** Double 3 is 6 so 3 + 4 is one more	Count back on a number line or number track  **ABSTRACT** 8 – 2 = 6 (NC- read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs)

SUBTRACTION: Y2

Understanding the operation and vocabulary

Understanding the operation

Understand subtraction as:

- taking away
- comparison (finding the difference)
- partitioning a set

Show that subtraction of one number from another cannot be done in any order

Recognise that 5 - 3 is different from 3 - 5

Recognise the inverse relationship between addition and subtraction

Write the related number sentences

5 + 2 = 7 2 + 5 = 7 7 = 5 + 2 7 = 2 + 5
7 – 2 = 5 7 – 5 = 2 2 = 7 - 5 5 = 7 - 2

Solve missing number problems e.g.

27 - ☐ = 17 ☐ = 21 - 4 10 = ☐ - ☐

Vocabulary

Understand the vocabulary related to subtraction
Also see Y1

subtraction, subtract, take away, difference, difference between, minus, tens, ones, partition, near multiple of 10, tens boundary, less than, one less, two less… ten less… one hundred less, more, one more, two more... ten more... one hundred more

Generalisation

Noticing what happens when you count back in tens (the digits in the ones column stay the same)
odd – odd = even; odd – even = odd; etc
Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Number Sense and Fluency

Number facts

Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100 e.g.
15 subtract 8 4 less than 12 80 minus 30
90 take 50

Know complements to the next multiple of 10 e.g.
52 + ☐ = 60 52 + ☐ = 80

Know pairs of multiples of 10 with a total of 100 and derive related subtraction facts e.g.
100 - 10, 100 - 20, 100 - 30 …

Mental methods and jottings

Subtract numbers using concrete objects, pictorial representations, and mentally, including:

* a two-digit number and ones
* a two-digit number and tens
* two two-digit numbers

Counting back in ones, twos and tens

57 – 20 (by counting back in tens; 47, 37)

With jottings

57 – 23 (by partitioning the second number and counting back; -20, -3)

Counting up

31 - 28 (by counting up from 28 by bridging the tens boundary; +2, +1)

With jottings

65 – 47 (by counting up from 47 by bridging the tens boundary; +3, +10, +5)

Adjusting

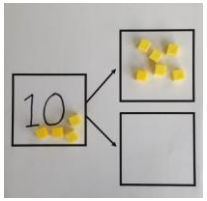
35 – 9 (by subtracting 10 and adding 1)

Recording

https://www.ncetm.org.uk/resources/50640

CONCRETE

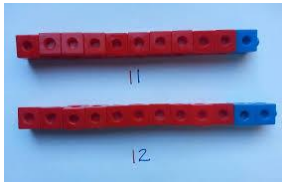
10 - 6 =



Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

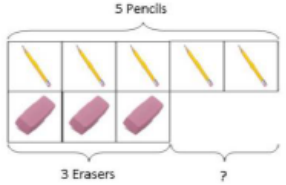
If 10 is the whole and 6 is one of the parts. What is the other part?

Compare amounts and objects to find the difference.

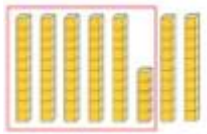


Use cubes to build towers or make bars to find the difference

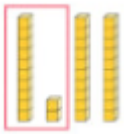
Use basic bar models with items to find the difference

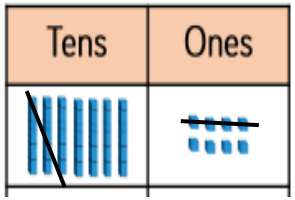


75 – 20



32 – 20



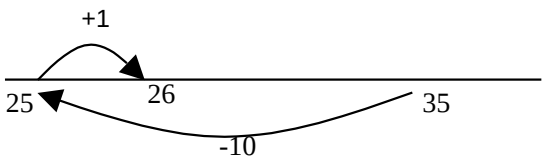


78 – 34

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.

Misconceptions

- Pupils struggle to interpret whether to add or subtract from the language used
- Pupils often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this.
- When subtracting, pupils will sometimes subtract the larger number from the smaller initially.
- When counting back, pupils may start counting using the start number itself rather than counting the next number.
- The equals sign is not always correctly interpreted as 'has the same value as' by pupils who may see it as 'the answer is'.
- Pupils do not use place value when adding and subtracting - signs of this can be them counting repeatedly from 0 or failing to use models that group tens differently. They may need to use a wider range of representations to develop this idea more strongly - some pupils can do this when the model 'looks' like 10 but not when the visual link has gone.
- Pupils may not always understand that addition is commutative but subtraction is not.



With jottings

35 –19 (by subtracting 20 and adding 1)

Using known facts and Place Value

57 – 4 (7 – 4 = 3 so 57 – 4 = 53)

Estimating

check calculations by subtracting in a different way

solve 16 – 9 by 16 – 10 + 1

Partitioning second number

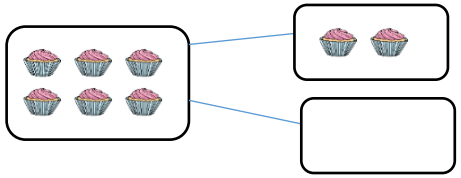
partition numbers in different ways

23 = 20 + 3 and 23 = 10 + 13 to support subtraction

57 – 23 (-20, -3) then cross tens boundary: 42 – 17 (-10, -2, -5)

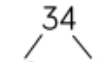
PICTORIAL

Use a pictorial representation of objects to show the part-part-whole model.



ABSTRACT

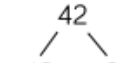
34 – 13 = ____



–10 –3

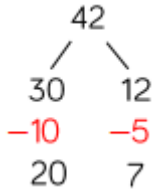
20 1

42 – 15 =



–10 –5

We can't subtract the ones here so need to partition differently. This relies on secure place value knowledge.



Now we can subtract the ones and then the tens

SUBTRACTION: Y3

Understanding the operation and vocabulary

Understanding the operation

Continue to develop understanding of subtraction

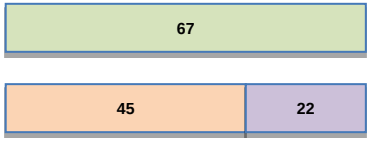
Use larger numbers to at least 1,000 and practise partitioning in different ways in preparation for written subtraction methods e.g.

146 = 100 + 40 + 6, 146 = 130 + 16

Understand that the principles of the commutative and associative laws do not apply to subtraction

Recognise that 41 - 35 is different from 35 - 41

Understand the inverse relationship between addition and subtraction



Write the related number sentences

45 + 22 = 67 22 + 45 = 67 67 = 45 + 22 67 = 22 + 45

67 – 22 = 45 67 – 45 = 22 22 = 67 - 45 45 = 67 - 22

Solve missing number problems e.g.

62 - ☐ = 19 ☐ = 68 - 54 ☐ - ☐ = 25

59 + 34 = 100 - ☐ 45 < ☐ - 6 ☐ - ☐ > 54 + 9

Vocabulary

Understand, read and spell vocabulary related to subtraction.

Also see Y1 and Y2

17 - 9 = 8

minuend – subtrahend = difference

Number Sense and Fluency

Number facts

Continue to recall and use subtraction facts to 20 fluently, and derive and use related facts beyond 100 e.g.

16 subtract 9, 150 minus 70, the difference between 80 and 170, 30 fewer than 110

Know pairs of two-digit numbers with a total of 100 and derive related subtraction facts e.g.

100 - 79, 100 - 43, 100 - 12 ….

Use knowledge of number bonds to 10 and 100 to calculate with multiples of 10

120 – 90 using knowledge of 12-9

Use knowledge of place value to subtract to or from a multiple of 10

90 – 27, 164 – 40 (count on/ back in tens)

Mental methods and jottings

subtract numbers mentally, including:

* a three-digit number and ones
* a three-digit number and tens
* a three-digit number and hundreds

Counting Back (sequencing)

164 – 40 (by counting back in tens; 154, 144, 134, 124)

387 – 59 (- 60 +1)

With Jottings:

375 – 47 (by partitioning the second number and counting back; -40, -5, -2) using a number line, 100 square or jottings

Counting up

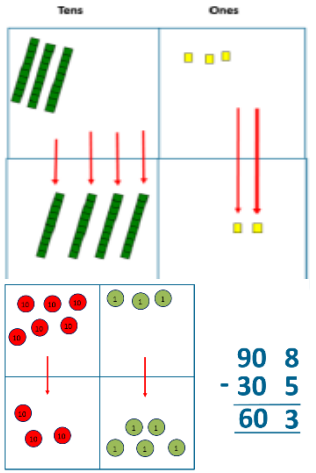
102 – 97 (by counting up from 97, bridging the hundreds boundary; +3, +2)

Recording

https://www.ncetm.org.uk/resources/50640

Subtraction without exchanging

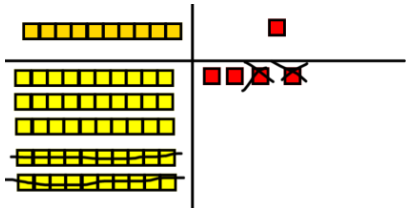
CONCRETE



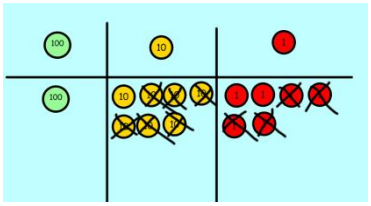
Use Diennes to make the larger number then physically subtract the smaller number.

90 8
- 30 5
60 3

PICTORIAL



54 – 22 =



176 – 64 =

hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.
odd – odd = even etc (see Year 2)
Inverses and related facts – develop fluency in finding related addition and subtraction facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Misconceptions

- Pupils struggle to interpret whether to add or subtract from the language used.
- When adding/subtracting 1s, 10s or 100s mentally, pupils may 'change' the digit in the wrong column.
- When subtracting, pupils may subtract the larger number from the smaller initially.
- When performing columnar subtraction, pupils may exchange from the wrong column or fail to exchange altogether (instead just finding the difference between the digits in the column, even where the second one is greater than the first).
- Pupils may also fail to correctly record the exchange and thus not reduce the tens, for example, by one so that the answer is 10 too high.
- Pupils often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this.
- When working with addition and subtraction facts, pupils sometime realise there is a connection e.g. 3 + 4 = 7 but then incorrectly rearrange this to make a false second fact e.g. 4 + 7 = 3.
- This is particularly true with subtraction facts, where pupils struggle to place the numbers in a correct order.
- Some pupils may use the incorrect operation when checking and fail to realise that they need to use

With jottings
343 – 170 (by counting up from 170, bridging the hundreds boundary; +30, +100, +43)

Adjusting:
234 – 99 (by subtracting 100 and adding 1)

With Jottings:
387 – 59 (by subtracting 60 and adding 1)

Using Known Facts And Place Value:
268 – 5
8 – 5 = 3 so 268 – 5 = 263

Estimating
Estimate the answer to a calculation
163 - 48 is approximately 150 - 50

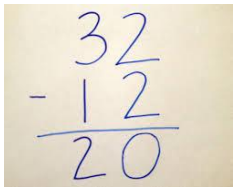
Use inverse operations to check answers
check 102 – 97 = 5 with 97 + 5 = 102

Use equivalent calculations to check answers

ABSTRACT

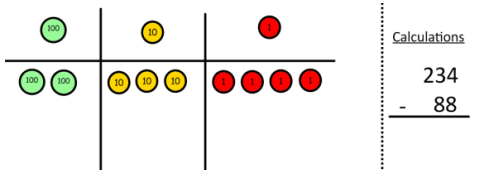
47 – 24 =

	40	7
-	20	4
	20	3

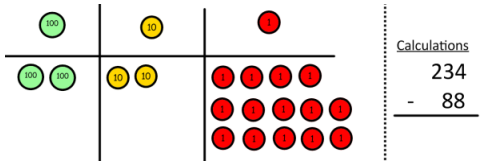


Subtraction with exchanging

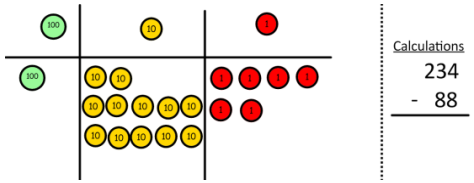
CONCRETE

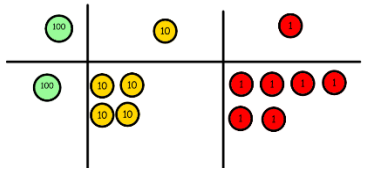


Exchange one of the 10s for ten 1s

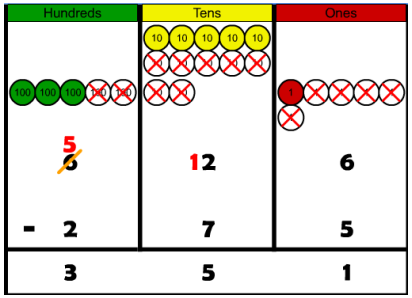


After subtracting the ones, exchange one of the 100s for ten 10s





Now calculation can be completed.

the inverse - this is more pronounced when subtracting. • Pupils struggle to add and subtract from right to left in columns and hence may end up with answers that are not partitioned into hundreds, tens and ones. • Pupils may place the smallest number at the top of the calculation when using column subtraction method. • When numbers exchanges happen, pupils may forget to notate them and hence not include the extra/fewer tens, hundreds etc. in the new calculations. • The equals sign is not always correctly interpreted as 'has the same value as' by pupils who may see it as 'the answer is'.		**PICTORIAL**  **ABSTRACT** Expanded Method H T O 200 30 40 1 7 - 100 20 9 100 10 8 = 118 Compact Method 2 3 4 7 - 1 2 9 1 1 8

SUBTRACTION: Y4

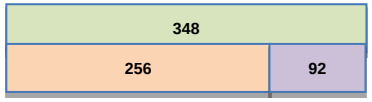
Understanding the operation and vocabulary

Understanding the operation

Continue to understand that the principles of the commutative and associative laws do not apply to subtraction

Recognise that 92 - 56 is different from 56 - 92

Continue to understand the inverse relationship between addition and subtraction



Write the related number sentences

256 + 92 = 348 92 + 256 = 348

348 = 256 + 92 348 = 92 + 256

348 – 256 = 92 348 – 92 = 256

92 = 348 - 256 256 = 348 – 92

Continue to solve missing number problems e.g.

456 - ☐ = 210 ☐ = 300 - 176 ☐ - ☐ = 125

589 + 318 = 1000 - ☐ 450 < ☐ - 60 ☐ - ☐ > 345 + 199

Vocabulary

Understand, read and spell vocabulary related to subtraction.

Also see Y1 Y2 and Y3

17 - 9 = 8

minuend – subtrahend = difference

subtract, subtraction, difference, less, take away, decrease, fewer, minus, count on, partition, adjust, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? equals sign, is the same as.

Number Sense and Fluency

Number facts

Continue to use knowledge of subtraction facts and place value to derive related facts, including decimals and money e.g.

8000 subtract 3000, 1700 minus 800,

the difference between 700 and 1400,

300 fewer than 1200

Know complements to the next multiple of 100 e.g.

367 + ☐ = 400 739 + ☐ = 800

Mental methods and jottings

Continue to practise mental methods of subtraction with increasingly large numbers.

Counting Back (Sequencing):

564 – 150 (by partitioning the second number and counting back; -100, -50)

With Jottings:

732 – 137 (by partitioning the second number and counting back; -100, -32, -5)

Counting Up:

607 – 288 (by counting up from 288, bridging the hundreds boundary; +12, +7)

With Jottings:

6070 – 4987 (by counting up from 4987, bridging the thousands boundary; +13, +1070)

Adjusting:

1487 – 199 (by subtracting 200 and adding 1)

With Jottings:

442 – 79 (by subtracting 80 (-40, -40) and adding 1)

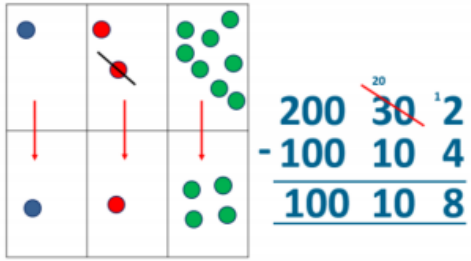
Using Known Facts And Place Value:

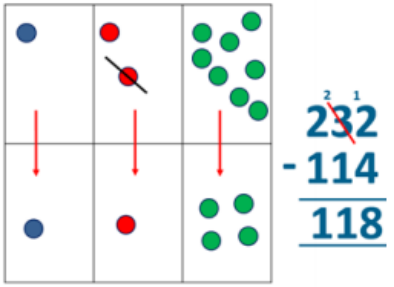
7000 - 600

Recording

https://www.ncetm.org.uk/resources/50640

CONCRETE TO PICTORIAL





ABSTRACT

Th	H	T	O
3000	200 300	160	7
- 1000	100	80	5
2000	100	80	2 = 1182

3 2 3 1 6 7
- 1 1 8 5
1 1 8 2

Generalisations

Investigate when re-ordering works as a strategy for subtraction. eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

Misconceptions

- Pupils struggle to interpret whether to add or subtract from the language used
- When subtracting, pupils will sometimes subtract the larger number from the smaller initially.
- When performing columnar subtraction, pupils may exchange from the wrong column or fail to exchange altogether (instead just finding the difference between the digits in the column, even where the second one is greater than the first).
- Pupils may also fail to correctly record the exchange and thus not reduce the tens, for example, by one so that the answer is 10 too high.
- Pupils find calculations where multiple exchanges must be made particularly hard e.g. 2304 - 1789 cause issues because of the need to carry out a chain reaction of exchange. In these instances you may need to resort back to equipment to secure understanding.
- Pupils often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this.
- Some pupils may use the incorrect operation when checking and fail to realise that they need to use the inverse - this is more pronounced when subtracting.

1000 – 600 = 400 so 7000 – 600 = 6400

Estimating:

Estimate the answer to a calculation

3062 - 2581 is approximately 3000 - 2500

Use inverse operations to check answers

check 564 – 150 = 414 with 414 + 150 = 564

Use equivalent calculations to check answers

Pupils will subtract decimals to 2 decimal places (in the context of money or measures)

£64.81 - £25.62 =

	£10	£1	.	10p	1p
	$^{5}\cancel{6}$	$^{1}4$	.	$^{7}\cancel{8}$	$^{1}1$
-	2	5	.	6	2
	£ 3	9	.	1	9

SUBTRACTION: Y5

Understanding the operation and vocabulary

Understanding the operation

Continue to solve missing number problems

6.5 - ☐ = 2.3   ☐ = 3 - 0.8   ☐ - ☐ = 1.2

5.4 + 2.7 = 10.3 - ☐   5.2 < ☐ - 0.9   ☐ - ☐ > 7.2 - 1.9

Begin to use brackets

(10 - 3) x 6 = ☐   10 - (0.5 x 7) = ☐

Use inverse operations and brackets

I am thinking of a number. I double it and then subtract 6. My answer is 8. What was my number?

☐ x 2 – 6 = 8 so (8 + 6) ÷ 2 = ☐

Vocabulary

Understand, read and spell vocabulary related to subtraction.

Also see previous years

17 - 9 = 8

minuend – subtrahend = difference

tens of thousands boundary,

Generalisation

Sometimes, always or never true?

The difference between a number and its reverse will be a multiple of 9.

What do you notice about the differences between consecutive square numbers?

Misconceptions

- Pupils struggle to interpret whether to add or subtract from the language used.
- Pupils can find 'How many more/less?' particularly troublesome as it relates to ordinal values of numbers and relationships.

Number Sense and Fluency

Number facts

Continue to use knowledge of subtraction facts and place value to derive related facts with numbers to one decimal place

1.2 subtract 0.7, 1.8 minus 0.9,

the difference between 2 and 1.3, 0.3 fewer than 1.7

Know complements to 1

0.78 + ☐ = 1   0.52 + ☐ = 1

Recall pairs of three-digit numbers with a total of 1000 and derive related subtraction facts

1000 - 453, 1000 - 239, 1000 - 712 …

Mental methods and jottings

Subtract numbers mentally with increasingly large numbers

Subtract tenths, and one-digit whole numbers and tenths

Counting Back (sequencing):

4.7 – 1.5 (by partitioning the second number and counting back; -1, -0.5)

With jottings:

19.2 – 2.7 (by partitioning the second number and counting back; -2, -0.2, -0.5)

Counting up:

7.2 – 6.8 (by counting up from 6.8 by bridging the units boundary; +0.2, +0.2)

With jottings:

8.3 – 4.8 (by counting up from 4.8 by bridging the units boundary; +0.2, +3.3)

Adjusting:

Recording

https://www.ncetm.org.uk/resources/50640

Pupils may still need the support of practical apparatus or use the expanded method initially.

Once confident using the compact method showing an understanding of the value of each digit, this method can be used for decimals too.

75.4 – 8.6

```
   60   14   1.
   ~~70~~   ~~5~~   0.4
-        8   0.6
   60    6   0.8   = 66.8
```

```
   6  14   1
   ~~7~~  ~~5~~ . 4
-     8 . 6
   6  6 . 8
```

• When subtracting, pupils will sometimes subtract the larger number from the smaller initially. • When performing columnar subtraction, pupils may exchange from the wrong column or fail to exchange altogether (instead just finding the difference between the digits in the column, even where the second one is greater than the first). • Pupils may also fail to correctly record the exchange and thus not reduce the tens, for example, by one so that the answer is 10 too high. • Pupils often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this.	8.3 – 1.9 (by subtracting 2 and adding 0.1) With jottings: 12.6 – 3.9 (by subtracting 4 and adding 0.1) **Using known facts and place value:** 15 – 0.3 1- 0.3 = 0.7 so 15 - 0.3 = 14.7 **Estimating** Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy 25 034 – 7185 is approximately 25 000 – 7000 Continue to use appropriate strategies to check answers check 4.7 – 1.5= 3.2 with 3.2 + 1.5	

SUBTRACTION: Y6

Understanding the operation and vocabulary

Understanding the operation

Use their knowledge of the order of operations

Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction

Understand that if the operations are at the same level of priority, work out the example from left to right

Continue to solve missing number problems

0.63 - ☐ = 0.32 ☐ = 0.5 - 0.33 ☐ - ☐ = 0.11

0.89 – 0.4 = 1.3 - ☐ 0.75 < ☐ - 0.06 ☐ - ☐ > 0.82 – 0.09

Explore the order of operations using brackets

compare 14 – (3 + 5) with (14 – 3) + 5

Use algebraic methods to solve missing number problems

2n +7 = 12 so 12 – 7 ÷ 2= n

Vocabulary

Understand, read and spell vocabulary related to subtraction correctly.

Also see previous years

17 - 9 = 8

minuend – subtrahend = difference

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Pupils could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering.

Sometimes, always or never true? Subtracting numbers makes them smaller.

Number Sense and Fluency

Number facts

Continue to use knowledge of subtraction facts and place value to derive related facts with numbers to two decimal places

3.09 subtract 0.04, 0.16 minus 0.08, the difference between 0.2 and 0.12, 0.06 fewer than 0-19

Know complements to the next whole number

4.83 + ☐ = 5 7.125 + ☐ = 8

Mental methods and jottings

Perform mental calculations, including with mixed operations, large numbers and decimals

Calculate intervals across zero e.g.

the drop in temperature from +5 to -3

Counting Back (Sequencing):

7.87 – 2.03 (by partitioning the second number and counting back; -2, -0.03)

With Jottings:

16.3 – 3.55 (by partitioning the second number and counting back; -3, -0.3, -0.25)

Counting up:

6.14 – 5.76 (by counting up from 5.76 by bridging the units boundary; +0.24, +0.14)

With Jottings:

8.3 – 4.54 (by counting up from 4.54 by bridging the units boundary; +0.46, +3.3)

Adjusting:

7.65 – 0.99 (by subtracting 1 and adding 0.01)

With Jottings:

15.4 – 3.09 (by subtracting 3.1 and adding 0.01)

Recording

https://www.ncetm.org.uk/resources/50640

By this stage, pupils should be confident using the compact method, for decimals up to 3 places.

500 203 – 34 456 63.15 – 19.76

5 12 10

~~6~~ ~~3~~ . ~~1~~ ^{1}5

- 1 9 . 7 6

4 3 . 3 9

Extend to decimals with different numbers of decimal places.

13.86 – 9.481

15

~~1~~^{1}3 . 8 ~~6~~^{1}0

- 9 . 4 8 1

4 . 4 7 9

Misconceptions • When subtracting pupils may subtract the larger number from the smaller initially. • When performing columnar subtraction pupils may exchange the wrong column or fail to exchange altogether. • When adding and subtracting numbers of different magnitude (including decimals of different lengths), pupils often misalign these in column addition and subtraction	**Using Known Facts And Place Value:** 1.63 – 0.8 16 – 8 = 8 so 1.63 - 0.8 = 0.83 **Estimating:** Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. 60 .31 – 17.884 is approximately 60 – 18 Continue to use appropriate strategies to check answers check 6.7 – 0.55 = 6.15 with 6.15 + 0.55	