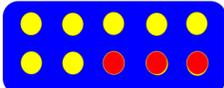
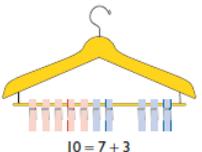
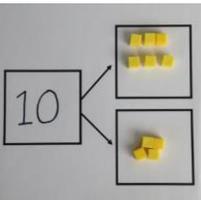
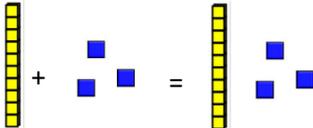


# ADDITION: Y1

Understanding the operation and vocabulary	Number Sense and Fluency	Recording
<p><b>Understanding the operation</b></p> <p>Understand addition as:</p> <ul style="list-style-type: none"> <li>- Increasing one quantity.</li> <li>- Combining two or more parts/quantities to make a whole.</li> </ul> <p>Read, write and interpret mathematical statements involving addition (+) and equals (=) sign.</p> <p><math>14 + 5 = 19</math>    <math>17 = 9 + 8</math></p> <p>Solve missing number problems</p> <p><math>11 + \square = 18</math>    <math>\square = 13 + 2</math>    <math>13 = \square + \square</math></p> <p>Understand addition and subtraction as related operations. e.g. <math>7 + 3 = 10</math> is related to <math>10 - 3 = 7</math></p> <p>When introduced to the equals sign, pupil should see it as signifying equality. They should become used to seeing it in different positions.</p> <p><b>Vocabulary</b></p> <p>Understand the vocabulary related to addition:</p> <p>addition, add (+), forwards, put together, more than, total, altogether, difference between, equals = same as, most, odd, even, digit, counting on, plus, the sum of</p> <p><b>Generalisations</b></p> <ul style="list-style-type: none"> <li>• True or false? Addition makes numbers bigger.</li> <li>• True or false? You can add numbers in any order and still get the same answer.</li> </ul> <p><b>Misconceptions</b></p> <ul style="list-style-type: none"> <li>• <math>\_\_ + 3 = 10</math> When pupils are faced with problems such as this they see two numbers and add them (e.g. <math>3 + 10 = 13</math>) instead of reading it as a sentence.</li> <li>• When working with number facts and bonds, pupil sometime realise there is a connection e.g. <math>3 + 4 = 7</math></li> </ul>	<p><b>Number facts</b></p> <p>Recall and use addition facts to 10 fluently</p> <p>the total of 6 and 3    6 plus 2    4 more than 5</p> <p>Know number pairs with a total of 20</p> <p><math>16 + \square = 20</math>    <math>20 = 3 + \square</math></p> <p><b>Mental methods and jottings</b></p> <p>Add one-digit and two-digit numbers to 20, including zero using concrete objects, pictorial representation and mentally.</p> <p>Represent and use number bonds within 20, experiencing the = sign in different positions.</p> <p><b>Counting on (sequencing)</b></p> <p><math>12 + 3</math> (by counting on in ones; 13, 14, 15)</p> <p><u>With Jottings:</u></p> <p>Progress to crossing the tens boundary</p> <p><math>18 + 5</math> (by partitioning 5 to bridge the tens boundary; <math>+ 2, + 3</math>)</p> <p><b>Partitioning</b></p> <p><math>5 + 7</math> ( by partitioning 7 in to 5 and 2)</p> <p><math>5 + 5 + 2</math></p> <p>Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop their understanding of place value.</p> <p>Pupils have opportunities to explore partitioning numbers in different ways.</p> <p>e.g. <math>7 = 6 + 1, 7 = 5 + 2, 7 = 4 + 3</math></p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p style="text-align: center;"><math>10 = 7 + 3</math></p>	<p><a href="https://www.ncetm.org.uk/resources/50640">https://www.ncetm.org.uk/resources/50640</a></p> <p><b>CONCRETE</b></p> <div style="display: flex; justify-content: space-around;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">  <div style="border: 1px solid black; padding: 5px;"> <p>Use cubes to add two numbers together as a group or in a bar.</p> </div> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;">  </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;">  </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 20px;">  </div>

but then incorrectly rearrange this to make a false second fact e.g.  $4 + 7 = 3$

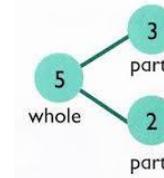
- Pupils struggle to interpret whether to add or subtract from the language used
- Pupils 'count on' to find the difference between their starting number and ten instead of using their number bonds to help them.
- Sometimes pupils count their starting number e.g. when finding the number pair  $6 + \underline{\quad} = 10$  they begin counting with the six and say '6, 7, 8, 9, 10' and therefore believe the missing number to be 5.
- The equals sign is not always correctly interpreted as 'has the same value as' by pupils, who may see it as 'the answer is'.

### Using known facts and place value

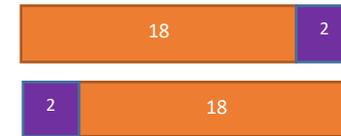
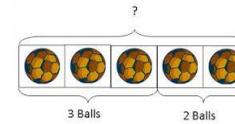
$$15 + 4$$

$$5 + 4 = 9 \text{ so } 15 + 4 = 19$$

### PICTORIAL



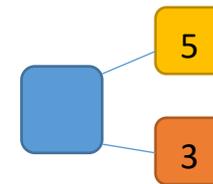
Use pictures to add two numbers together as a group or in a bar.



### ABSTRACT

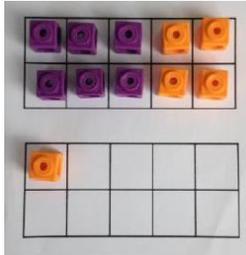
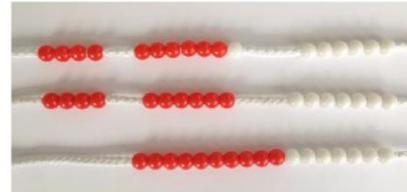
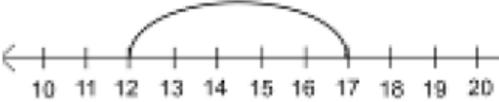
$$4 + 3 = 7$$

Use the part-part whole diagram to move into the abstract.



(NC- read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs)

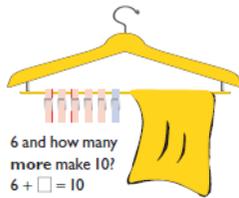
## ADDITION: Y2

Understanding the operation and vocabulary	Understanding the operation and vocabulary	Understanding the operation and vocabulary
<p><b>Understanding the operation</b> Continue to understand addition as:</p> <ul style="list-style-type: none"> <li>- Combining two or more quantities.</li> <li>- Increasing one quantity.</li> </ul> <p>Show that addition of two numbers can be done in any order (commutative law) <i>Recognise that <math>5 + 27</math> is equal to <math>27 + 5</math></i></p> <p>Continue to recognise the inverse relationship between addition and subtraction using numbers up to 20. <i>Write the related number sentences</i> <math>15 + 2 = 17</math>   <math>2 + 15 = 17</math>   <math>17 = 15 + 2</math>   <math>17 = 2 + 15</math> <math>17 - 2 = 15</math>   <math>17 - 15 = 2</math>   <math>2 = 17 - 15</math>   <math>15 = 17 - 2</math></p> <p>Solve missing number problems <math>17 + \square = 27</math>   <math>\square = 21 + 4</math>   <math>10 = \square + \square</math></p> <p><b>Vocabulary</b> Understand the vocabulary related to addition</p> <p>+, add, addition, more, plus, make, sum, total, altogether, how many more to make...? how many more is... than...? how much more is...? =, equals, sign, is the same as, tens, ones, partition near multiple of 10, tens boundary, more than, one more, two more... ten more... one hundred more</p> <p><b>Generalisation</b></p> <ul style="list-style-type: none"> <li>• Noticing what happens when you count in tens (the digits in the ones column stay the same)</li> <li>• odd + odd = even; odd + even = odd; etc</li> <li>• Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot</li> <li>• Recognise and use the inverse relationship between addition and subtraction and use this to check</li> </ul>	<p><b>Number facts</b> Recall and use number facts to 20 fluently and derive and use related facts up to 100. <i>7 add 8   4 more than 9   50 plus 30</i> <i>the sum of 40 and 50</i></p> <p>Know complements to the next multiple of 10. <math>52 + \square = 60</math>   <math>76 + \square = 80</math></p> <p>Know pairs of multiples of 10 with a total of 100. <math>60 + \square = 100</math>   <math>100 = 70 + \square</math></p> <p><b>Mental methods and jottings</b> Add numbers using concrete objects, pictorial representations, and mentally, including:</p> <ul style="list-style-type: none"> <li>* a two-digit number and ones</li> <li>* a two-digit number and tens</li> <li>* two two-digit numbers</li> <li>* adding three one-digit numbers</li> </ul> <p><b>Counting on</b> <math>37 + 20</math> (by counting on in tens; 47, 57)</p> <p><b>With Jottings</b> Begin by not crossing the tens boundary <math>42 + 23</math> (by partitioning the second number and counting on; + 20, + 3)</p> <p>Progress to crossing the tens boundary <math>47 + 15</math> (by partitioning the second number and counting on; + 10, +3, +2)</p> <p><b>Partitioning</b> <math>23 + 12</math> (<math>20 + 10 = 30</math>, <math>3 + 2 = 5</math> then <math>30 + 5 = 35</math>)</p> <p><b>With Jottings</b> Begin by not crossing the tens boundary <math>42 + 23</math> (<math>40 + 20 = 60</math>; <math>3 + 2 = 5</math> then <math>60 + 5</math>)</p>	<p><a href="https://www.ncetm.org.uk/resources/50640">https://www.ncetm.org.uk/resources/50640</a></p> <p><b>CONCRETE</b></p>  <p><b>PICTORIAL</b></p>  <p>Start with the bigger number and use the smaller number to make 10. <math>4 + 7 + 6 = 17</math> Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p> <p><math>12 + 5 = 17</math></p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>

calculations and missing number problems. This understanding could be supported by images such as this.



$$7 + ? = 10$$



#### Misconceptions

- Pupils struggle to interpret whether to add or subtract from the language used
- Pupils struggle to add two digit numbers when their place value understanding is weak. If they do not read a number like '52' as 5 tens and 2 ones then they struggle to combine the ones and the tens from two numbers appropriately.
- When counting on, pupil may start counting the first extra number using the start number itself, rather than counting the next number (thus they end up with one less than the real answer.
- Pupils do not use place value when adding and subtracting. Signs of this can be them counting repeatedly from 0 or failing to use models that group tens differently. They may need to use a wider range of representations to develop this idea more strongly - some pupil can do this when the model 'looks' like 10 but not when the visual link has gone.
- The equals sign is not always correctly interpreted as 'has the same value as' by pupils who may see it as 'the answer is'.

Progress to crossing the tens boundary  
 $47 + 15$  ( $40 + 10 = 50$ ,  $7 + 5 = 12$  then  $50 + 12 = 62$ )

#### Adjusting

$34 + 9$  (adding 10 then subtracting 1)

#### With Jottings

$45 + 19$  (by adding 20 and subtracting 1)

#### Using known facts and place value:

$63 + 4$

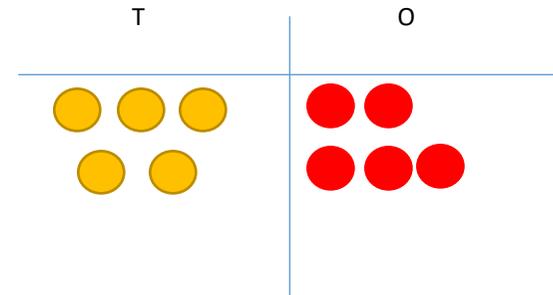
$3 + 4 = 7$  so  $63 + 4 = 67$

#### Estimating:

Check calculations by adding in a different order

check  $27 + 15$  ( $27 + 10 + 5$ ) with  $15 + 20 + 7$

After practically using the base 10 blocks and place value counters, pupil can draw the counters or base 10 to help them to solve additions.



#### ABSTRACT

Use of partitioning

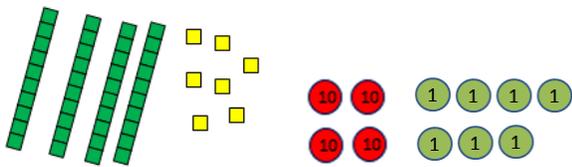
$$53 + 24 = 77$$

$$50 + 20 = 70$$

$$3 + 4 = 7$$

$$70 + 7 = 77$$

## ADDITION: Y3

Understanding the operation and vocabulary	Number Sense and Fluency	Recording																				
<p><b>Understanding the operation</b> Understand the principles of the commutative and associative law: Recognise that <math>45 + 36</math> is equal to <math>36 + 45</math> Recognise that if calculating <math>13 + 14 + 9</math> the numbers can be combined in any order</p> <p>Understand the inverse relationship between addition and subtraction <math>45 + 22 = 67</math>   <math>22 + 45 = 67</math>   <math>67 = 45 + 22</math>   <math>67 = 22 + 45</math> <math>67 - 22 = 45</math>   <math>67 - 45 = 22</math>   <math>22 = 67 - 45</math>   <math>45 = 67 - 22</math></p> <p>Solve missing number problems <math>62 + \square = 74</math>   <math>\square = 45 + 32</math>   <math>\square + \square = 50</math> <math>100 - 3 = 67 + \square</math>   <math>45 &lt; \square + 6</math>   <math>\square + \square &gt; 54 + 9</math></p> <p><b>Vocabulary</b> Understand, read and spell vocabulary related to addition correctly Also see Y1 and Y2</p> <p style="text-align: center;"><math>8 + 9 = 17</math> addend + addend = sum</p> <p>hundreds, tens, ones, estimate, partition, recombine, difference, decrease, increase, near multiple of 10 and 100, inverse, rounding, column, exchange, complements, &lt; and &gt;</p> <p><b>Generalisations</b> Noticing what happens to the digits when you count in tens and hundreds. odd + odd = even etc (see Year 2) Inverses and related facts – develop fluency in finding related addition and subtraction facts. Develop the knowledge that the inverse relationship can be used as a checking method.</p>	<p><b>Number facts</b> Continue to recall and use addition facts to 20 fluently, and derive and use related facts beyond 100 <math>7 \text{ add } 9</math>, <math>80 \text{ plus } 70</math>, the sum of 90 and 60, 30 more than 110</p> <p>Know pairs of two-digit numbers with a total of 100 <math>74 + \square = 100</math>   <math>100 = 59 + \square</math></p> <p>Pupil need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.</p> <p><b>Mental methods and jottings</b> Add numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds</p> <p>Continue to use number lines and number squares to support mental strategies</p> <p>Manipulatives can be used to support mental imagery and conceptual understanding. Pupil need to be shown how these images are related eg.</p> <p>What's the same? What's different?</p> <div style="text-align: center;">  </div> <p><b>Counting On (Sequencing)</b> <math>137 + 50</math> (by counting on in tens; 147, 157, 167, 177, 187)</p>	<p><a href="https://www.ncetm.org.uk/resources/50640">https://www.ncetm.org.uk/resources/50640</a></p> <p><b>Addition without re-grouping</b></p> <p><b>CONCRETE</b></p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> <table style="width: 100%; text-align: center;"> <tr><td>T</td><td>O</td></tr> <tr><td>■■■■■</td><td>■■■■■</td></tr> <tr><td>■■■■■</td><td>■■■■■</td></tr> <tr><td>■■■■■</td><td>■■■■■</td></tr> </table> </div> <div style="border: 1px solid black; padding: 5px;"> <table style="width: 100%; text-align: center;"> <tr><td>10</td><td>1</td></tr> <tr><td>10 10 10 10</td><td>1 1 1 1</td></tr> <tr><td>10</td><td>1 1 1 1 1</td></tr> </table> </div> </div> <p>Add the ones first and then the tens using Diennes and place value counters.</p> <p><b>PICTORIAL</b></p> <div style="display: flex; align-items: center;"> <table style="border-collapse: collapse; margin-right: 20px;"> <tr><td style="text-align: center;">T</td><td style="border-left: 1px solid black; padding-left: 10px; text-align: center;">O</td></tr> <tr><td style="text-align: center;">● ● ●</td><td style="border-left: 1px solid black; padding-left: 10px; text-align: center;">● ●</td></tr> <tr><td style="text-align: center;">● ●</td><td style="border-left: 1px solid black; padding-left: 10px; text-align: center;">● ● ●</td></tr> </table> <p>Pupils can draw the counters to represent the counters</p> </div> <p><b>ABSTRACT</b></p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> <math display="block">\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}</math> </div>	T	O	■■■■■	■■■■■	■■■■■	■■■■■	■■■■■	■■■■■	10	1	10 10 10 10	1 1 1 1	10	1 1 1 1 1	T	O	● ● ●	● ●	● ●	● ● ●
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### Misconceptions

- Pupils struggle to interpret whether to add or subtract from the language used.
- Pupils struggle to add two and three digit numbers when their place value understanding is weak. If they do not read a number like '352' as 3 hundreds, 5 tens and 2 ones then they struggle to combine the ones, tens and the hundreds from two numbers appropriately.
- When adding/subtracting 1s, 10s or 100s mentally, pupils may 'change' the digit in the wrong column.
- When performing columnar addition, pupils may forget to include the tens or hundreds they have generated from earlier exchanges. They may also fail to exchange them at all and thus end with a two-digit numbers in the ones column.
- When working with addition and subtraction facts, pupils sometime realise there is a connection e.g.  $3 + 4 = 7$  but then incorrectly rearrange this to make a false second fact e.g.  $4 + 7 = 3$ .
- Some pupils may use the incorrect operation when checking and fail to realise that they need to use the inverse - this is more pronounced when subtracting.
- When attempting missing number problems e.g.  $245 + ? = 300$  pupil may give the answer 65 as they have counted up in tens then ones
- The equals sign is not always correctly interpreted as 'has the same value as' by pupils, who may see it as 'the answer is'.
- Pupils may not add from right to left in columns and hence may not have answers that are partitioned into hundreds, tens and ones.

### With Jottings:

$345 + 37$  (by partitioning the second number and counting on;  $+30, +5, +2$ )

### Partitioning the second number:

$$236 + 33 \text{ (} 236 + 30 + 3 \text{)}$$

### With Jottings and practical equipment:

$$247 + 125 = 247 + 100 + 20 + 5$$

$$= 347 + 20 + 5$$

$$= 367 + 5$$

$$= 372$$

Use manipulatives to support partitioning :

$$236 + 85 = 236 + 80 + 5$$

$$= 236 + 70 + 10 + 4 + 1$$

### Adjusting:

$$234 + 99 \text{ (by adding 100 and subtracting 1)}$$

### With Jottings:

$$334 + 59 \text{ (by adding 60 and subtracting 1)}$$

### Using near doubles:

$$18 + 16 = \text{double } 18 - 2 \text{ or double } 16 + 2$$

### Using Known Facts And Place Value:

$$282 + 7$$

$$2 + 7 = 9 \text{ so } 282 + 7 = 289$$

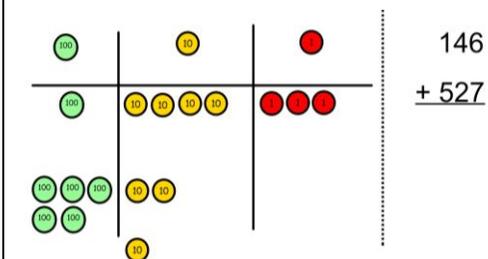
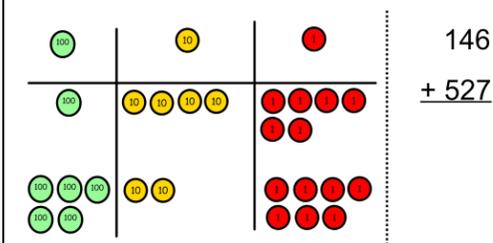
### Estimating:

Estimate the answer to a calculation

$$139 + 58 \text{ is approximately } 150 + 50$$

### Addition with re-grouping

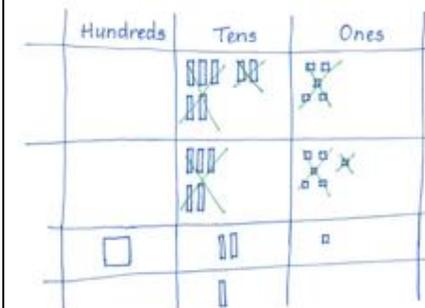
#### CONCRETE



Add the 1s and exchange ten 1s for one 10

Pupils should also experience exchanging ten 10s for one 100.

#### PICTORIAL



**Checking:**

Use inverse operations /equivalent calculations to check

236 + 85 by adding in a different order

e.g. 200 + 85 + 36

Check answers by adding in a different order

**ABSTRACT****Expanded method**

$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{O} \\
 200 \quad 40 \quad 7 \\
 + 100 \quad 20 \quad 5 \\
 \hline
 300 \quad 60 \quad 12 = 372
 \end{array}$$

...leading to

$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 200 \quad 40 \quad 7 \\
 + 100 \quad 20 \quad 5 \\
 \hline
 300 \quad 70 \quad 2 = 372
 \end{array}$$

10

**Compact method**

$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 2 \quad 4 \quad 7 \\
 + 1 \quad 2 \quad 5 \\
 \hline
 3 \quad 7 \quad 2
 \end{array}$$

1

**Expanded Column Addition**

1. Write the numbers so that the digits with the same place value are lined up like this.

2. Start by adding the ones together then write the answer below the answer line.

3. Add the tens together and write the answer on a new line.

4. Add the hundreds together and write the answer on a new line.

5. Add up the three lines in the answer box.

## ADDITION: Y4

Understanding the operation and vocabulary	Number Sense and Fluency	Recording																																																	
<p><b>Understanding the operation</b></p> <p>Continue to understand the principles of the commutative and associative laws</p> <p style="color: green;">Recognise that <math>342 + 187</math> is equal to <math>187 + 342</math></p> <p style="color: green;">Recognise that if calculating <math>46 + 39 + 14</math> the numbers can be combined in any order</p> <p>Continue to understand the inverse relationship between addition and subtraction</p> <p style="color: green;"><math>256 + 92 = 348</math>    <math>92 + 256 = 348</math>    <math>348 = 256 + 92</math>  <math>348 = 92 + 256</math>    <math>348 - 256 = 92</math>    <math>348 - 92 = 256</math>  <math>92 = 348 - 256</math>    <math>256 = 348 - 92</math></p> <p>Continue to solve missing number problems and understand that these can often be solved by working backwards</p> <p style="color: green;"><math>456 + \square = 673</math>    <math>\square = 300 + 176</math>    <math>\square + \square = 125</math>  <math>1000 - 103 = 450 + \square</math>    <math>450 &lt; \square + 60</math>  <math>\square + \square &gt; 345 + 199</math></p> <p><b>Vocabulary</b></p> <p>Understand, read and spell vocabulary related to addition correctly</p> <p>Also see Years 1, 2 and 3</p> <p style="text-align: center;"><math>8 + 9 = 17</math></p> <p>addend + addend = sum</p> <p>add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make...? how much more? units boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? equals sign, is the same as, exchange, bridge, adjust.</p>	<p><b>Number facts</b></p> <p>Continue to use knowledge of addition facts and place value to derive related facts</p> <p style="color: green;">5000 add 3000, 700 plus 800, the sum of 700 and 600, 300 more than 1200</p> <p>Know complements to the next multiple of 100</p> <p style="color: green;"><math>568 + \square = 600</math>    <math>749 + \square = 800</math></p> <p>Continue to practise mental methods of addition with increasingly large numbers.</p> <p>Add multiples of 10, 100 and 1000, using manipulatives such as Dienes or place value counters to support. Begin to extend this to decimals</p> <p>Use number lines to reinforce decimals being between whole numbers</p> <p><b>Mental methods and jottings</b></p> <p><b>Counting On (Sequencing):</b></p> <p style="color: green;"><math>534 + 150</math> (partitioning the second number and counting on; +100, +50)</p> <p><u>With Jottings:</u></p> <p style="color: green;"><math>675 + 28</math> (by partitioning the second number and counting on; +25, +3)</p> <p><b>Partitioning:</b></p> <p style="color: green;"><math>87 + 46</math> (<math>80 + 40 = 120</math>, <math>7 + 6 = 13</math>, <math>120 + 13 = 133</math>)</p> <p><u>With Jottings:</u></p> <p style="color: green;"><math>456 + 362</math> (<math>400 + 300 = 700</math>, <math>50 + 60 = 110</math>, <math>6 + 2 = 8</math>, <math>700 + 110 + 8 = 818</math>)</p>	<p><a href="https://www.ncetm.org.uk/resources/50640">https://www.ncetm.org.uk/resources/50640</a></p> <p>Pupils will add decimals to 2 decimal places (in the context of money or measures)</p> <p style="color: red;"><b>CONCRETE</b></p> <p style="color: black;"><math>247 + 125</math></p> <div style="border: 1px solid black; 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**Misconceptions**

- Pupils struggle to interpret whether to add or subtract from the language used.
- Pupils struggle to add numbers when their place value understanding is weak. If they do not read a number like '4352' as 4 thousands, 3 hundreds, 5 tens and 2 ones then they struggle to combine the ones, tens, hundreds and thousands from two numbers appropriately.
- When performing columnar addition, pupils may forget to include the hundreds, tens or hundreds they have generated from earlier exchanges.
- They may also fail to exchange them at all and thus end with a two-digit numbers in the ones column etc.
- Pupils find calculations where multiple exchanges must be made particularly hard e.g. 4678 + 3945 because the notation becomes unwieldy.
- The equals sign is not always correctly interpreted as 'has the same value as' by pupils who may see it as 'the answer is'.

**Adjusting:**

1435 + 199 (by adding 200 and subtracting 1)

With Jottings:

1764 + 79 (by adding 80 (+40, +40) and subtracting 1)

Using near doubles:

36 + 35 = one more than 70

Using Known Facts And Place Value:

6060 + 47

60 + 47 = 107 so 6060 + 47 = 6107

14 + 15 = 29 so 140 + 150 = 290

Re-ordering calculations:

Investigate when re-ordering works as a strategy

e.g. 46 + 39 + 14 = 46 + 14 + 39

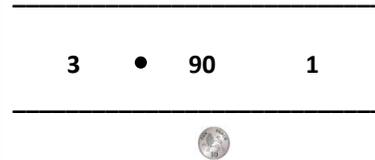
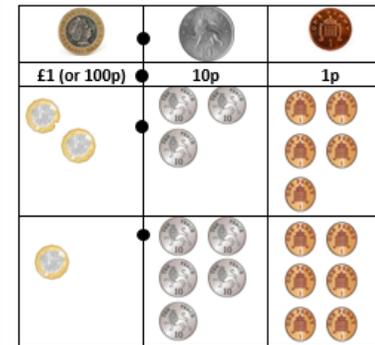
Estimating:

Estimate the answer to a calculation

2467 + 1729 is approximately 2500 + 1500

Use inverse operation or an equivalent calculations to check answers

1764 + 79 by adding 80 and adjusting or by using partitioning



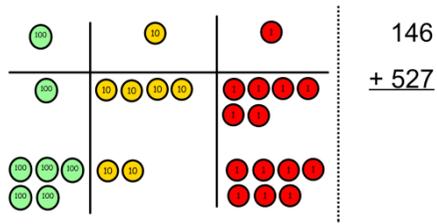
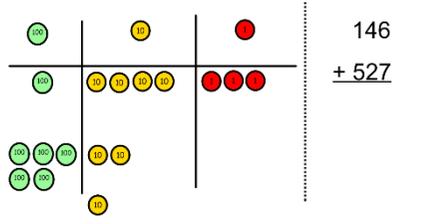
**ABSTRACT**

Th	H	T	O
3000	800	40	7
<u>2000</u>	<u>300</u>	<u>20</u>	<u>5</u>
6000	100	70	2 = 6,172
1000		10	

Th	H	T	O
3	8	4	7
+ 2	3	2	5
<u>6</u>	<u>1</u>	<u>7</u>	<u>2</u>
1		1	

	£	2	3	.	5	9
+	£		7	.	5	5
<hr/>						
	£	3	1	.	1	4
		1	1		1	

## ADDITION: Y5

Understanding the operation and vocabulary	Number sense and fluency	Recording
<p><b>Understanding the operation</b> Continue to solve missing number problems  <math>6.5 + \square = 10.7</math>   <math>\square = 8.4 + 3.7</math>   <math>\square + \square = 4.2</math>  <math>7.3 + 2.9 = 9.9 + \square</math>   <math>5.2 &lt; \square - 0.9</math>   <math>\square - \square &gt; 7.2 - 1.9</math></p> <p>Begin to use brackets  <math>(10+3) \times 7 = \square</math>   <math>\square = 10 + (0.4 \times 8)</math></p> <p><b>Vocabulary</b> Understand, read and spell mathematical vocabulary related to addition correctly Also see previous years</p> <p>tens of thousands boundary,</p> <p style="text-align: center;"><math>8 + 9 = 17</math> addend + addend = sum</p> <p><b>Generalisation</b> Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers?</p> <p><b>Misconceptions</b></p> <ul style="list-style-type: none"> <li>Pupils struggle with the different concepts of the magnitude of a number and the sign of a number e.g. they think that -6 is greater than 3. It is important that they understand that 'greater' means 'higher up the number line'</li> <li>Pupils confuse the meaning of &lt; and &gt;, finding it hard to tell which is which.</li> <li>When counting in powers of 10, pupils struggle when bridging 10, 100 etc e.g. they think that <math>997 + 100 = 1197</math> and forget about 1097.</li> </ul>	<p><b>Number facts</b> Continue to use knowledge of addition facts and place value to derive related facts with numbers to one decimal place  1.2 plus 0.7, the total of 0.8 and 0.9, the sum of 0.2 and 1.3, 0.3 more than 1.7</p> <p>Know complements to 1  <math>0.78 + \square = 1</math>   <math>0.52 + \square = 1</math></p> <p>Recall pairs of three-digit numbers with a total of 1000  <math>456 + \square = 1000</math>   <math>1000 = \square + 825</math></p> <p><b>Mental methods and jottings</b> Add numbers mentally with increasingly large numbers. Add tenths, and one-digit whole numbers and tenths.</p> <p><b>Counting on (sequencing):</b>  <math>4.3 + 1.5</math> (by partitioning the second number and counting on; +1, +0.5)</p> <p><b>With jottings:</b>  <math>19.7 + 2.6</math> (by partitioning the second number and counting on; +2, +0.3, +0.3)</p> <p><b>Partitioning:</b>  <math>3.6 + 1.7</math> (<math>3 + 1 = 4</math>, <math>0.6 + 0.7 = 1.3</math>, <math>4 + 1.3 = 5.3</math>)</p> <p><b>With jottings:</b>  <math>18.7 + 14.8</math>  <math>(18 + 14 = 32, 0.7 + 0.8 = 1.5, 32 + 1.5 = 33.5)</math></p> <p><b>Adjusting:</b>  <math>8.3 + 1.9</math> (by adding 2 and subtracting 0.1)</p>	<p><a href="https://www.ncetm.org.uk/resources/50640">https://www.ncetm.org.uk/resources/50640</a></p> <p>Add whole numbers with more than 4 digits, including using formal written methods</p> <p><b>CONCRETE</b> Place value counters, dienes</p> <p>Make both numbers on a place value grid.</p> <div style="text-align: right;">  <p style="margin-right: 20px;"><math>146</math> <math>+ 527</math></p> </div> <p>Add up the units and exchange 10 ones for one 10.</p> <div style="text-align: right;">  <p style="margin-right: 20px;"><math>146</math> <math>+ 527</math></p> </div> <p>Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.</p> <p>This can also be done with Base 10 to help pupil clearly see that 10 ones equal 1 ten and 10 tens equal 100.</p> <p>As pupil move on to decimals, money and decimal place value counters can be used to support learning.</p>

- Pupils struggle to interpret whether to add or subtract from the language used.
- Pupils struggle to add numbers when their place value understanding is weak. If they do not read a number like '4352' as 4 thousands, 3 hundreds, 5 tens and 2 ones then they struggle to combine the ones, tens, hundreds and thousands from two numbers appropriately.
- When performing columnar addition, pupils may forget to include the hundreds tens or hundreds they have generated from earlier exchanges.
- They may also fail to exchange them at all and thus end with a two-digit numbers in the 1s column etc.

With jottings:

$$14.6 + 3.9 \text{ (by adding 4 and subtracting 0.1)}$$

**Using known facts and place value:**

$$7.5 + 2.6$$

$$7.5 + 2.5 = 10 \text{ so } 7.5 + 2.6 = 10.1$$

**Estimating**

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

$$25\,063 + 7459 \text{ is approximately } 25\,000 + 7500$$

Continue to use appropriate strategies to check answers

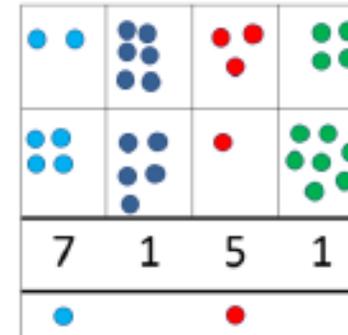
check  $8.3 + 1.9$  by adding in a different order

$$8.3 + 2 - 0.1 \text{ or } 8.3 + 0.7 + 1.2$$

## PICTORIAL

Bar model, part/part/whole, place value counters, number lines.

Pupil can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



## ABSTRACT

**Written methods (progressing to more than 4-digits)**

As year 4, progressing when understanding of the expanded method is secure, pupil will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

$$\begin{array}{r} 25063 \\ + 7459 \\ \hline 32522 \\ \hline \end{array}$$

$$\begin{array}{r} 172.83 \\ + 54.68 \\ \hline 227.51 \\ \hline \end{array}$$

$$\begin{array}{r} 23.361 \\ + 9.080 \\ \hline 59.770 \\ + 1.300 \\ \hline 93.511 \\ \hline \end{array}$$

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

## ADDITION: Y6

Understanding the operation and vocabulary	Number sense and fluency	Recording
<p><b>Understanding the operation</b> Use their knowledge of the order of operations.</p> <p>Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction.</p> <p>Understand that if the operations are at the same level of priority, work out the example from left to right.</p> <p>Continue to solve missing number problems</p> <p><math>0.63 + \square = 0.85</math>   <math>\square = 0.5 + 0.33</math>   <math>\square + \square = 0.71</math></p> <p><math>0.89 + 0.3 = 0.6 + \square</math>   <math>0.75 &lt; \square + 0.06</math></p> <p><math>\square + \square &gt; 0.74 + 0.07</math></p> <p>Explore the order of operations using brackets compare <math>14 - (3 + 5)</math> with <math>(14 - 3) + 5</math></p> <p><b>Vocabulary</b> See previous years</p> <p>Understand, read and spell mathematical vocabulary related to addition correctly</p> <p style="text-align: center;"><math>8 + 9 = 17</math></p> <p>addend + addend = sum</p> <p><b>Generalisations</b> Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Pupil could learn an acrostic such as BODMAS/BIDMAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller.</p>	<p><b>Number facts</b> Continue to use knowledge of addition facts and place value to derive related facts with numbers to two decimal places</p> <p>0.09 plus 0.04, the total of 0.09 and 0.08, the sum of 0.06 and 0.12 , 0.04 more than 1.13</p> <p>Know complements to the next whole number</p> <p><math>4.83 + \square = 5</math>   <math>7.125 + \square = 8</math></p> <p><b>Mental methods and jottings</b> Perform mental calculations, including with mixed operations, large numbers and decimals</p> <p>Add positive and negative integers (in contexts such as temperature) a <math>6^{\circ}\text{C}</math> temperature rise from <math>-4^{\circ}\text{C}</math></p> <p><b>Counting On (Sequencing):</b> <math>6.46 + 2.03</math> (by partitioning the second number and counting on; +2, +0.03)</p> <p><b>With Jottings:</b> <math>18.7 + 5.64</math> (by partitioning the second number and counting on; +5, +0.3, +0.34)</p> <p><b>Partitioning:</b> <math>3.4 + 2.77</math> (<math>3+2=5</math>, <math>0.4+0.7=1.1</math>, <math>5+1.1+0.07=6.17</math>)</p> <p><b>With Jottings:</b> <math>27.34 + 5.78</math> (<math>27 + 5 = 32</math>, <math>0.3 + 0.7 = 1</math>, <math>0.04 + 0.08 = 0.12</math>, <math>33 + 1 + 0.12 = 34.12</math>)</p> <p><b>Adjusting:</b> <math>6.73 + 0.99</math> (by adding 1 and subtracting 0.01)</p>	<p><a href="https://www.ncetm.org.uk/resources/50640">https://www.ncetm.org.uk/resources/50640</a></p> <p><b>Concrete, pictorial, written</b> (see Year 5 above)</p> <p><b>Written methods</b> As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.</p> <p>Continue calculating with decimals, including those with different numbers of decimal places</p> <p><b><math>13.86 + 9.481 = 23.341</math></b></p> $  \begin{array}{r}  13.860 \\  + 9.481 \\  \hline  23.341 \\  1\ 1\ 1  \end{array}  $

**Misconceptions**

- When adding and subtracting numbers of different magnitude (including decimals of different lengths), pupil often misalign these in column addition.

**With Jottings:**

17.4 + 5.09 (by adding 5.1 and subtracting 0.01)

**Using Known Facts And Place Value:**

0.64 + 0.36

64 + 36 = 100 so 0.64 + 0.36 = 1

**Estimating:**

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

73.82 + 17.382 is approximately 74 + 17

Continue to use appropriate strategies to check answers

check 3.4 + 2.77 by adding in a different order  
partition or add 3 and adjust